

Paper Reference(s)

**6666/01**

# Edexcel GCE

## Core Mathematics C4

### Gold Level (Harder) G3

**Time: 1 hour 30 minutes****Materials required for examination**

Mathematical Formulae (Green)

**Items included with question papers**

Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

**Instructions to Candidates**

---

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C4), the paper reference (6666), your surname, initials and signature.

**Information for Candidates**

---

A booklet 'Mathematical Formulae and Statistical Tables' is provided.

Full marks may be obtained for answers to ALL questions.

There are 8 questions in this question paper. The total mark for this paper is 75.

**Advice to Candidates**

---

You must ensure that your answers to parts of questions are clearly labelled.

You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

**Suggested grade boundaries for this paper:**

<b>A*</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>	<b>E</b>
<b>65</b>	<b>58</b>	<b>47</b>	<b>42</b>	<b>36</b>	<b>28</b>

1.  $f(x) = (3 + 2x)^{-3}, \quad |x| < \frac{3}{2}.$

Find the binomial expansion of  $f(x)$ , in ascending powers of  $x$ , as far as the term in  $x^3$ .

Give each coefficient as a simplified fraction.

(5)

June 2007

---

2. Using the substitution  $u = \cos x + 1$ , or otherwise, show that

$$\int_0^{\frac{\pi}{2}} e^{\cos x + 1} \sin x \, dx = e(e - 1).$$

(6)

June 2010

---

3. Express  $\frac{9x^2 + 20x - 10}{(x + 2)(3x - 1)}$  in partial fractions.

(4)

January 2013

---

4. With respect to a fixed origin  $O$  the lines  $l_1$  and  $l_2$  are given by the equations

$$l_1 : \mathbf{r} = \begin{pmatrix} 11 \\ 2 \\ 17 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix} \quad l_2 : \mathbf{r} = \begin{pmatrix} -5 \\ 11 \\ p \end{pmatrix} + \mu \begin{pmatrix} q \\ 2 \\ 2 \end{pmatrix}$$

where  $\lambda$  and  $\mu$  are parameters and  $p$  and  $q$  are constants. Given that  $l_1$  and  $l_2$  are perpendicular,

- (a) show that  $q = -3$ .

(2)

Given further that  $l_1$  and  $l_2$  intersect, find

- (b) the value of  $p$ ,

(6)

- (c) the coordinates of the point of intersection.

(2)

The point  $A$  lies on  $l_1$  and has position vector  $\begin{pmatrix} 9 \\ 3 \\ 13 \end{pmatrix}$ . The point  $C$  lies on  $l_2$ .

Given that a circle, with centre  $C$ , cuts the line  $l_1$  at the points  $A$  and  $B$ ,

- (d) find the position vector of  $B$ .

(3)

**January 2009**

---

5.

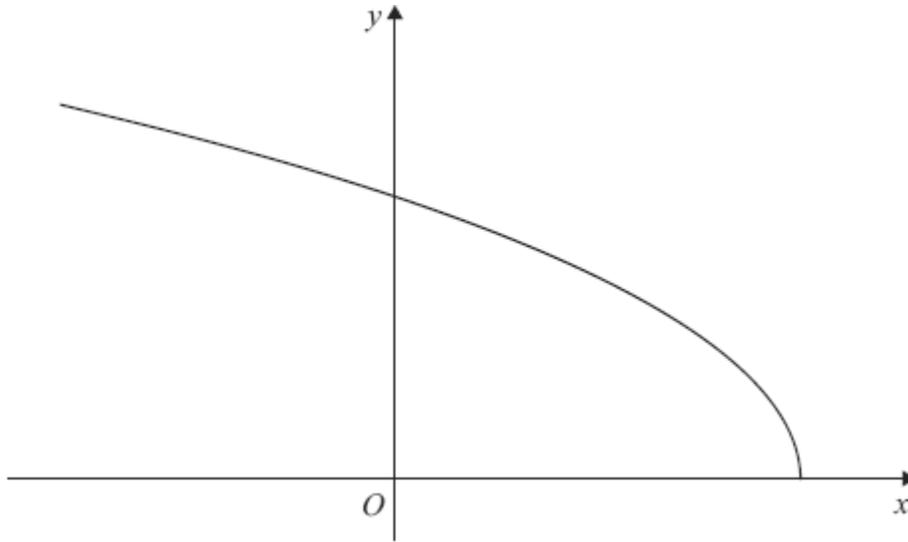
**Figure 2**

Figure 2 shows a sketch of the curve with parametric equations

$$x = 2 \cos 2t, \quad y = 6 \sin t, \quad 0 \leq t \leq \frac{\pi}{2}.$$

(a) Find the gradient of the curve at the point where  $t = \frac{\pi}{3}$ .

**(4)**

(b) Find a cartesian equation of the curve in the form

$$y = f(x), \quad -k \leq x \leq k,$$

stating the value of the constant  $k$ .

**(4)**

(c) Write down the range of  $f(x)$ .

**(2)****January 2009**

6. The points  $A$  and  $B$  have position vectors  $2\mathbf{i} + 6\mathbf{j} - \mathbf{k}$  and  $3\mathbf{i} + 4\mathbf{j} + \mathbf{k}$  respectively.

The line  $l_1$  passes through the points  $A$  and  $B$ .

(a) Find the vector  $\overrightarrow{AB}$ . (2)

(b) Find a vector equation for the line  $l_1$ . (2)

A second line  $l_2$  passes through the origin and is parallel to the vector  $\mathbf{i} + \mathbf{k}$ . The line  $l_1$  meets the line  $l_2$  at the point  $C$ .

(c) Find the acute angle between  $l_1$  and  $l_2$ . (3)

(d) Find the position vector of the point  $C$ . (4)

**January 2008**

---

7. The line  $l_1$  has equation  $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ , where  $\lambda$  is a scalar parameter.

The line  $l_2$  has equation  $\mathbf{r} = \begin{pmatrix} 0 \\ 9 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix}$ , where  $\mu$  is a scalar parameter.

Given that  $l_1$  and  $l_2$  meet at the point  $C$ , find

(a) the coordinates of  $C$ . (3)

The point  $A$  is the point on  $l_1$  where  $\lambda = 0$  and the point  $B$  is the point on  $l_2$  where  $\mu = -1$ .

(b) Find the size of the angle  $ACB$ . Give your answer in degrees to 2 decimal places. (4)

(c) Hence, or otherwise, find the area of the triangle  $ABC$ . (5)

**June 2010**

---

8. A population growth is modelled by the differential equation

$$\frac{dP}{dt} = kP,$$

where  $P$  is the population,  $t$  is the time measured in days and  $k$  is a positive constant.

Given that the initial population is  $P_0$ ,

- (a) solve the differential equation, giving  $P$  in terms of  $P_0$ ,  $k$  and  $t$ .

(4)

Given also that  $k = 2.5$ ,

- (b) find the time taken, to the nearest minute, for the population to reach  $2P_0$ .

(3)

In an improved model the differential equation is given as

$$\frac{dP}{dt} = \lambda P \cos \lambda t,$$

where  $P$  is the population,  $t$  is the time measured in days and  $\lambda$  is a positive constant.

Given, again, that the initial population is  $P_0$  and that time is measured in days,

- (c) solve the second differential equation, giving  $P$  in terms of  $P_0$ ,  $\lambda$  and  $t$ .

(4)

Given also that  $\lambda = 2.5$ ,

- (d) find the time taken, to the nearest minute, for the population to reach  $2P_0$  for the first time, using the improved model.

(3)

**June 2007**

---

**TOTAL FOR PAPER: 75 MARKS**

**END**

Question Number	Scheme	Marks
1. (a)	<p>** represents a constant</p> $f(x) = (3 + 2x)^{-3} = \underline{(3)^{-3}} \left(1 + \frac{2x}{3}\right)^{-3} = \frac{1}{\underline{27}} \left(1 + \frac{2x}{3}\right)^{-3}$ $= \frac{1}{27} \left\{ 1 + (-3)(**x) + \frac{(-3)(-4)}{2!} (**x)^2 + \frac{(-3)(-4)(-5)}{3!} (**x)^3 + \dots \right\}$ <p><b>with <math>** \neq 1</math></b></p> $= \frac{1}{27} \left\{ 1 + (-3)\left(\frac{2x}{3}\right) + \frac{(-3)(-4)}{2!} \left(\frac{2x}{3}\right)^2 + \frac{(-3)(-4)(-5)}{3!} \left(\frac{2x}{3}\right)^3 + \dots \right\}$ $= \frac{1}{27} \left\{ 1 - 2x + \frac{8x^2}{3} - \frac{80}{27}x^3 + \dots \right\}$ $= \frac{1}{27} - \frac{2x}{27} + \frac{8x^2}{81} - \frac{80x^3}{729} + \dots$	<p>Takes 3 outside the bracket to give any of <math>(3)^{-3}</math> or <math>\frac{1}{27}</math>.  <b>See note below.</b></p> <p>Expands <math>(1 + **x)^{-3}</math> to give a simplified or an un-simplified <math>1 + (-3)(**x)</math>;  M1;</p> <p>A correct simplified or an un-simplified <math>\{ \dots \}</math> expansion  A1 ✓  with candidate's followed thro' <math>(**x)</math></p> <p>Anything that cancels to <math>\frac{1}{27} - \frac{2x}{27}</math>;  A1;  Simplified <math>\frac{8x^2}{81} - \frac{80x^3}{729}</math>  A1</p> <p style="text-align: right;"><b>[5]</b></p> <p style="text-align: right;"><b>5 marks</b></p>

2.	$\frac{du}{dx} = -\sin x$ $\int \sin x e^{\cos x+1} dx = -\int e^u du$ $= -e^u$ $= -e^{\cos x+1}$ $\left[-e^{\cos x+1}\right]_0^{\frac{\pi}{2}} = -e^1 - (-e^2)$ $= e(e-1) *$	<p>B1</p> <p>M1 A1</p> <p>A1ft</p> <p>ft sign error</p> <p>or equivalent with <math>u</math></p> <p>M1</p> <p>A1</p> <p>cso</p>	<p><b>(6)</b></p> <p><b>[6]</b></p>
----	---	---	-------------------------------------

Question Number	Scheme	Marks	
3.	$\frac{9x^2 + 20x - 10}{(x+2)(3x-1)} \equiv A + \frac{B}{x+2} + \frac{C}{3x-1}$ $A = 3$ $9x^2 + 20x - 10 \equiv A(x+2)(3x-1) + B(3x-1) + C(x+2)$ <p>Either <math>x^2: 9 = 3A, \quad x: 20 = 5A + 3B + C</math>                      constant: <math>-10 = -2A - B + 2C</math></p> <p>or</p> $x = -2 \Rightarrow 36 - 40 - 10 = -7B \Rightarrow -14 = -7B \Rightarrow B = 2$ $x = \frac{1}{3} \Rightarrow 1 + \frac{20}{3} - 10 = \frac{7}{3}C \Rightarrow -\frac{7}{3} = \frac{7}{3}C \Rightarrow C = -1$	<p>their constant term = 3</p> <p>Forming a correct identity.</p> <p>Attempts to find the value of either one of their <math>B</math> or their <math>C</math> from their identity.</p> <p>Correct values for their <math>B</math> and their <math>C</math>, which are found using a correct identity.</p>	<p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p>

**[4]**

Question Number	Scheme	Marks
<p>4 (a) <math>\mathbf{d}_1 = -2\mathbf{i} + \mathbf{j} - 4\mathbf{k}</math> , <math>\mathbf{d}_2 = q\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}</math></p> <p>As <math>\left\{ \mathbf{d}_1 \bullet \mathbf{d}_2 = \begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix} \bullet \begin{pmatrix} q \\ 2 \\ 2 \end{pmatrix} \right\} = \underline{(-2 \times q) + (1 \times 2) + (-4 \times 2)}</math></p> <p><math>\mathbf{d}_1 \bullet \mathbf{d}_2 = 0 \Rightarrow -2q + 2 - 8 = 0</math>  <math>-2q = 6 \Rightarrow \underline{q = -3}</math> AG</p> <p>(b) Lines meet where:</p> <p><math>\begin{pmatrix} 11 \\ 2 \\ 17 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix} = \begin{pmatrix} -5 \\ 11 \\ p \end{pmatrix} + \mu \begin{pmatrix} q \\ 2 \\ 2 \end{pmatrix}</math></p> <p><b>i:</b> <math>11 - 2\lambda = -5 + q\mu</math> (1)  <b>First two of j:</b> <math>2 + \lambda = 11 + 2\mu</math> (2)  <b>k:</b> <math>17 - 4\lambda = p + 2\mu</math> (3)</p> <p><b>(1) + 2(2)</b> gives: <math>15 = 17 + \mu \Rightarrow \mu = -2</math></p> <p><b>(2)</b> gives: <math>2 + \lambda = 11 - 4 \Rightarrow \lambda = 5</math></p> <p><b>(3)</b> <math>\Rightarrow 17 - 4(5) = p + 2(-2)</math>  <math>\Rightarrow p = 17 - 20 + 4 \Rightarrow \underline{p = 1}</math></p> <p>(c) <math>\mathbf{r} = \begin{pmatrix} 11 \\ 2 \\ 17 \end{pmatrix} + 5 \begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix}</math> or <math>\mathbf{r} = \begin{pmatrix} -5 \\ 11 \\ 1 \end{pmatrix} - 2 \begin{pmatrix} -3 \\ 2 \\ 2 \end{pmatrix}</math></p> <p>Intersect at <math>\mathbf{r} = \begin{pmatrix} 1 \\ 7 \\ -3 \end{pmatrix}</math> or <math>\underline{(1, 7, -3)}</math></p>	<p>Apply dot product calculation between two direction vectors, ie.  <math>\underline{(-2 \times q) + (1 \times 2) + (-4 \times 2)}</math></p> <p>Sets <math>\mathbf{d}_1 \bullet \mathbf{d}_2 = 0</math> and solves to find <math>\underline{q = -3}</math></p> <p>Need to see equations (1) and (2).          Condone one slip.          (Note that <math>q = -3</math>.)</p> <p>Attempts to solve (1) and (2) to find one of either <math>\lambda</math> or <math>\mu</math>          Any one of <math>\underline{\lambda = 5}</math> or <math>\underline{\mu = -2}</math>          Both <math>\underline{\lambda = 5}</math> and <math>\underline{\mu = -2}</math></p> <p>Attempt to substitute their <math>\lambda</math> and <math>\mu</math> into their <b>k</b> component to give an equation in <math>p</math> alone.  <math>\underline{p = 1}</math></p> <p>Substitutes their value of <math>\lambda</math> or <math>\mu</math> into the correct line <math>l_1</math> or <math>l_2</math>.</p> <p><math>\begin{pmatrix} 1 \\ 7 \\ -3 \end{pmatrix}</math> or <math>\underline{(1, 7, -3)}</math></p>	<p>M1          A1 cso          (2)</p> <p>M1          dM1          A1          A1          ddM1          A1          cso          (6)</p> <p>M1          A1          (2)</p>

Question Number	Scheme	Marks
(d)	<p>Let <math>\overline{OX} = \mathbf{i} + 7\mathbf{j} - 3\mathbf{k}</math> be point of intersection</p> $\overline{AX} = \overline{OX} - \overline{OA} = \begin{pmatrix} 1 \\ 7 \\ -3 \end{pmatrix} - \begin{pmatrix} 9 \\ 3 \\ 13 \end{pmatrix} = \begin{pmatrix} -8 \\ 4 \\ -16 \end{pmatrix}$ <p>Finding vector <math>\overline{AX}</math> by finding the difference between <math>\overline{OX}</math> and <math>\overline{OA}</math>. Can be ft using candidate's <math>\overline{OX}</math>.</p> $\overline{OB} = \overline{OA} + \overline{AB} = \overline{OA} + 2\overline{AX}$ $\overline{OB} = \begin{pmatrix} 9 \\ 3 \\ 13 \end{pmatrix} + 2 \begin{pmatrix} -8 \\ 4 \\ -16 \end{pmatrix} \qquad \begin{pmatrix} 9 \\ 3 \\ 13 \end{pmatrix} + 2 \left( \text{their } \overline{AX} \right)$ <p>Hence, <math>\overline{OB} = \begin{pmatrix} -7 \\ 11 \\ -19 \end{pmatrix}</math> or <math>\overline{OB} = \underline{-7\mathbf{i} + 11\mathbf{j} - 19\mathbf{k}}</math> <math>\begin{pmatrix} -7 \\ 11 \\ -19 \end{pmatrix}</math> or <math>\underline{-7\mathbf{i} + 11\mathbf{j} - 19\mathbf{k}}</math> or <math>\underline{(-7, 11, -19)}</math></p>	<p>M1 <math>\sqrt{\pm}</math></p> <p>dM1 <math>\sqrt{\phantom{x}}</math></p> <p>A1</p> <p>(3)</p> <p><b>[13]</b></p>

5. (a)	$\frac{dx}{dt} = -4 \sin 2t, \quad \frac{dy}{dt} = 6 \cos t$ $\frac{dy}{dx} = -\frac{6 \cos t}{4 \sin 2t} \quad \left( = -\frac{3}{4 \sin t} \right)$ <p>At <math>t = \frac{\pi}{3}</math>, <math>m = -\frac{3}{4 \times \frac{\sqrt{3}}{2}} = -\frac{\sqrt{3}}{2}</math> accept equivalents, awrt <math>-0.87</math></p>	<p>B1, B1</p> <p>M1</p> <p>A1 (4)</p>
(b)	<p>Use of <math>\cos 2t = 1 - 2 \sin^2 t</math></p> $\cos 2t = \frac{x}{2}, \quad \sin t = \frac{y}{6}$ $\frac{x}{2} = 1 - 2 \left( \frac{y}{6} \right)^2$ <p>Leading to <math>y = \sqrt{(18 - 9x)} \quad (= 3\sqrt{(2 - x)})</math> cao</p> $-2 \leq x \leq 2 \qquad k = 2$	<p>M1</p> <p>M1</p> <p>A1</p> <p>B1 (4)</p>
(c)	$0 \leq f(x) \leq 6 \qquad \text{either } 0 \leq f(x) \text{ or } f(x) \leq 6$ <p>Fully correct. Accept <math>0 \leq y \leq 6, [0, 6]</math></p>	<p>B1</p> <p>B1 (2)</p> <p><b>(10 marks)</b></p>

Question Number	Scheme	Marks
<p>6. (a)</p> <p>(b)</p> <p>(c)</p>	$\overline{OA} = \begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} \quad \& \quad \overline{OB} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix}$ $\overline{AB} = \overline{OB} - \overline{OA} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} - \begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$ $l_1: \mathbf{r} = \begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} \quad \text{or} \quad \mathbf{r} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$ $l_2: \mathbf{r} = \begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix} \quad \text{or} \quad \mathbf{r} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix}$ $l_3: \mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \Rightarrow \mathbf{r} = \mu \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ <p><math>\overline{AB} = \mathbf{d}_1 = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k}</math>, <math>\mathbf{d}_2 = \mathbf{i} + 0\mathbf{j} + \mathbf{k}</math> &amp; <math>\theta</math> is angle</p> $\cos \theta = \frac{\overline{AB} \cdot \mathbf{d}_2}{( \overline{AB}  \cdot  \mathbf{d}_2 )} = \frac{\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}}{(\sqrt{(1)^2 + (-2)^2 + (2)^2} \cdot \sqrt{(1)^2 + (0)^2 + (1)^2})}$ $\cos \theta = \frac{1+0+2}{\sqrt{(1)^2 + (-2)^2 + (2)^2} \cdot \sqrt{(1)^2 + (0)^2 + (1)^2}}$ $\cos \theta = \frac{3}{3\sqrt{2}} \Rightarrow \theta = 45^\circ \text{ or } \frac{\pi}{4} \text{ or awrt } 0.79.$	<p>Finding the difference between <math>\overline{OB}</math> and <math>\overline{OA}</math>. Correct answer.</p> <p>An expression of the form (vector) <math>\pm \lambda</math>(vector) <math>\mathbf{r} = \overline{OA} \pm \lambda</math>(their <math>\overline{AB}</math>) or <math>\mathbf{r} = \overline{OB} \pm \lambda</math>(their <math>\overline{AB}</math>) or <math>\mathbf{r} = \overline{OA} \pm \lambda</math>(their <math>\overline{BA}</math>) or <math>\mathbf{r} = \overline{OB} \pm \lambda</math>(their <math>\overline{BA}</math>) (<math>\mathbf{r}</math> is needed.)</p> <p>Considers dot product between <math>\mathbf{d}_2</math> and their <math>\overline{AB}</math>.</p> <p>Correct followed through expression or <b>equation</b>.</p> <p><math>\theta = 45^\circ</math> or <math>\frac{\pi}{4}</math> or awrt 0.79</p> <p>M1 <math>\pm</math> A1 [2]</p> <p>M1 A1 <math>\sqrt</math> aef [2]</p> <p>M1 <math>\sqrt</math></p> <p>A1 <math>\sqrt</math></p> <p>A1 <b>cao</b> [3]</p>

This means that  $\cos \theta$  does not necessarily have to be the subject of the equation. It could be of the form  $3\sqrt{2} \cos \theta = 3$ .

Question Number	Scheme	Marks
6. (d)	<p>If <math>l_1</math> and <math>l_2</math> intersect then: <math>\begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} = \mu \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}</math></p> <p><b>i:</b> <math>2 + \lambda = \mu</math> (1)  <b>j:</b> <math>6 - 2\lambda = 0</math> (2)  <b>k:</b> <math>-1 + 2\lambda = \mu</math> (3)</p> <p>(2) yields <math>\lambda = 3</math>  Any two yields <math>\lambda = 3, \mu = 5</math></p> <p><math>l_1: \mathbf{r} = \begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} + 3 \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix}</math> or <math>\mathbf{r} = 5 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix}</math></p>	<p><b>Either</b> seeing equation (2) written down correctly with or without any other equation <b>or</b> seeing equations (1) and (3) written down correctly. M1 <math>\sqrt{\phantom{x}}</math></p> <p>Attempt to solve either equation (2) or simultaneously solve any two of the three equations to find ... dM1</p> <p>either one of <math>\lambda</math> or <math>\mu</math> correct. A1</p> <p><math>\begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix}</math> or <math>5\mathbf{i} + 5\mathbf{k}</math> A1 <b>cs0</b></p> <p>Fully correct solution &amp; no incorrect values of <math>\lambda</math> or <math>\mu</math> seen earlier.</p> <p>[4]</p>

Question	Scheme	Marks
----------	--------	-------

<p><b>7.</b></p>	<p>(a) <b>j</b> components <math>3 + 2\lambda = 9 \Rightarrow \lambda = 3</math> <span style="float: right;"><math>(\mu = 1)</math></span>                      Leading to <math>C : (5, 9, -1)</math> <span style="float: right;">accept vector forms</span></p> <p>(b) <span style="float: right;">Choosing correct directions or finding <math>\overrightarrow{AC}</math> and <math>\overrightarrow{BC}</math></span>  <math display="block">\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix} = 5 + 2 = \sqrt{6} \sqrt{29} \cos \angle ACB</math> <span style="float: right;">use of scalar product</span>  <math>\angle ACB = 57.95^\circ</math> <span style="float: right;">awrt <math>57.95^\circ</math></span></p> <p>(c) <math>A : (2, 3, -4)</math> <math>B : (-5, 9, -5)</math>  <math display="block">\overrightarrow{AC} = \begin{pmatrix} 3 \\ 6 \\ 3 \end{pmatrix}, \quad \overrightarrow{BC} = \begin{pmatrix} 10 \\ 0 \\ 4 \end{pmatrix}</math> <math display="block">AC^2 = 3^2 + 6^2 + 3^2 \Rightarrow AC = 3\sqrt{6}</math> <math display="block">BC^2 = 10^2 + 4^2 \Rightarrow BC = 2\sqrt{29}</math> <math display="block">\Delta ABC = \frac{1}{2} AC \times BC \sin \angle ACB</math> <math display="block">= \frac{1}{2} 3\sqrt{6} \times 2\sqrt{29} \sin \angle ACB \approx 33.5</math> <span style="float: right;"><math>15\sqrt{5}</math>, awrt 34</span></p>	<p>M1 A1 A1 <b>(3)</b></p> <p>M1 M1 A1 A1 <b>(4)</b></p> <p>M1 A1 A1 M1 A1 <b>(5)</b></p> <p style="text-align: right;"><b>[12]</b></p>
------------------	---	---

Question Number	Scheme PhysicsAndMathsTutor.com	Marks
Number		
8. (c)	$\frac{dP}{dt} = \lambda P \cos \lambda t \quad \text{and} \quad t=0, P = P_0 \quad (1)$	
8. (a)	$\frac{dP}{dt} = kP \quad \text{and} \quad t=0, P = P_0 \quad (1)$  $\int \frac{dP}{P} = \int k dt$  $\ln P = kt; (+ c)$  When $t=0, P = P_0 \Rightarrow \ln P_0 = c$ (or $P = Ae^{kt} \Rightarrow P_0 = A$ )  $\ln P = kt + \ln P_0 \Rightarrow e^{\ln P} = e^{kt + \ln P_0} = e^{kt} \cdot e^{\ln P_0}$  Hence, $\underline{P = P_0 e^{kt}}$	  Separates the variables with $\int \frac{dP}{P}$ and $\int k dt$ on either side with integral signs not necessary. M1  Must see $\ln P$ and $kt$ ; Correct equation with/without $+ c$ . A1  Use of boundary condition (1) to attempt to find the constant of integration. M1  $\underline{P = P_0 e^{kt}}$ A1  [4]  (b) $P = 2P_0$ & $k = 2.5 \Rightarrow \underline{2P_0 = P_0 e^{2.5t}}$ M1  $e^{2.5t} = 2 \Rightarrow \underline{\ln e^{2.5t} = \ln 2}$ or $\underline{2.5t = \ln 2}$ ...or $e^{kt} = 2 \Rightarrow \underline{\ln e^{kt} = \ln 2}$ or $\underline{kt = \ln 2}$ M1  Eliminates $P_0$ and takes $\ln$ of both sides  $\Rightarrow t = \frac{1}{2.5} \ln 2 = 0.277258872... \text{ days}$  $t = 0.277258872... \times 24 \times 60 = 399.252776... \text{ minutes}$  $t = \underline{399 \text{ min}}$ or $t = \underline{6 \text{ hr } 39 \text{ mins}}$ (to nearest minute) A1  awrt $t = \underline{399}$ or $\underline{6 \text{ hr } 39 \text{ mins}}$  [3]

	$\int \frac{dP}{P} = \int \lambda \cos \lambda t \, dt$ <p><math>\ln P = \sin \lambda t; (+ c)</math></p> <p>When <math>t=0, P = P_0 \Rightarrow \ln P_0 = c</math> (or <math>P = Ae^{\sin \lambda t} \Rightarrow P_0 = A</math>)</p> <p><math>\ln P = \sin \lambda t + \ln P_0 \Rightarrow e^{\ln P} = e^{\sin \lambda t + \ln P_0} = e^{\sin \lambda t} \cdot e^{\ln P_0}</math></p> <p>Hence, <u><math>P = P_0 e^{\sin \lambda t}</math></u></p> <p>(d) <math>P = 2P_0</math> &amp; <math>\lambda = 2.5 \Rightarrow 2P_0 = P_0 e^{\sin 2.5t}</math></p> <p><math>e^{\sin 2.5t} = 2 \Rightarrow \underline{\sin 2.5t = \ln 2}</math> ...or ... <math>e^{\lambda t} = 2 \Rightarrow \underline{\sin \lambda t = \ln 2}</math></p> <p><u><math>t = \frac{1}{2.5} \sin^{-1}(\ln 2)</math></u></p> <p><math>t = 0.306338477\dots</math></p> <p><math>t = 0.306338477\dots \times 24 \times 60 = 441.1274082\dots</math> minutes</p> <p><math>t = \underline{441\text{min}}</math> or <math>t = \underline{7 \text{ hr } 21 \text{ mins}}</math> (to nearest minute)</p>	<p>Separates the variables with <math>\int \frac{dP}{P}</math> and <math>\int \lambda \cos \lambda t \, dt</math> on either side with integral signs not necessary.</p> <p>Must see <math>\ln P</math> and <math>\sin \lambda t</math>; Correct equation with/without + c.</p> <p>Use of boundary condition (1) to attempt to find the constant of integration.</p> <p><u><math>P = P_0 e^{\sin \lambda t}</math></u></p> <p>Eliminates <math>P_0</math> and makes <math>\sin \lambda t</math> or <math>\sin 2.5t</math> the subject by taking <math>\ln</math>'s</p> <p>Then rearranges to make <math>t</math> the subject. (must use <math>\sin^{-1}</math>)</p> <p>awrt <math>t = \underline{441}</math> or <u><math>7 \text{ hr } 21 \text{ mins}</math></u></p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>[4]</p> <p>M1</p> <p>dM1</p> <p>A1</p> <p>[3]</p> <p><b>14 marks</b></p>
--	---	--	---

**Question 1**

The majority of candidates produced correct solutions to this question, but a substantial minority of candidates were unable to carry out the first step of writing  $(3+2x)^{-3}$  as  $\frac{1}{27}\left(1+\frac{2x}{3}\right)^{-2}$ . Those who were able to do this could usually complete the remainder of the question but some sign errors and manipulation errors were seen. Another common error was for candidates to apply  $\frac{n(n-1)}{2!}$  and/or  $\frac{n(n-1)(n-2)}{3!}$  in the third and fourth terms of their expansion.

**Question 2**

This question was generally well done and, helped by the printed answer, many produced fully correct answers. The commonest error was to omit the negative sign when differentiating  $\cos x + 1$ . The order of the limits gave some difficulty. Instead of the correct  $-\int_2^1 e^u du$ , an incorrect version  $-\int_1^2 e^u du$  was produced and the resulting expressions manipulated to the printed result and working like  $-(e^2 - e^1) = -e^2 + e^1 = e(e - 1)$  was not uncommon.

Some candidates got into serious difficulties when, through incorrect algebraic manipulation, they obtained  $-\int e^u \sin^2 x du$  instead of  $-\int e^u du$ . This led to expressions such as  $\int e^u (u^2 - 2u) du$  and the efforts to integrate this, either by parts twice or a further substitution, often ran to several supplementary sheets. The time lost here inevitably led to difficulties in finishing the paper. Candidates need to have some idea of the amount of work and time appropriate to a 6 mark question and, if they find themselves exceeding this, realise that they have probably made a mistake and that they would be well advised to go on to another question.

**Question 3**

This was correctly answered by about 40% of the candidates.

A majority incorrectly expressed  $\frac{9x^2 + 20x - 10}{(x + 2)(3x - 1)}$  as  $\frac{2}{(x + 2)} - \frac{1}{(3x - 1)}$ , having failed to realise that the algebraic fraction given in the question is improper, thereby losing 3 of the 4 marks available.

For those achieving the correct partial fractions, a process of long division was typically used to find the value of the constant term, and the resulting remainder, usually  $5x - 4$ , became the LHS of the subsequent identity. A minority of them, however, applied  $9x^2 + 20x - 10 \equiv A(x + 2)(3x - 1) + B(3x - 1) + C(x + 2)$  in order to obtain the correct partial fractions.

### Question 4

The majority of candidates identified the need for some form of dot product calculation in part (a). Taking the dot product  $l_1.l_2$ , was common among candidates who did not correctly proceed, while others did not make any attempt at a calculation, being unable to identify the vectors required. A number of candidates attempted to equate  $l_1$  and  $l_2$  at this stage. The majority of candidates, however, were able to show that  $q = -3$ .

In part (b), the majority of candidates correctly equated the **i**, **j** and **k** components of  $l_1$  and  $l_2$ , and although some candidates made algebraic errors in solving the resulting simultaneous equations, most correctly found  $\lambda$  and  $\mu$ . In almost all such cases the value of  $p$  and the point of intersection in part (c) was then correctly determined.

There was a failure by many candidates to see the link between part (d) and the other three parts of this question with the majority of them leaving this part blank. Those candidates who decided to draw a diagram usually increased their chance of success. Most candidates who were successful at this part applied a vector approach as detailed in the mark scheme. The easiest vector approach, adopted by a few candidates, is to realise that  $\lambda = 1$  at  $A$ ,  $\lambda = 5$  at the point of intersection and so  $\lambda = 9$  at  $B$ . So substitution of  $\lambda = 9$  into  $l_1$  yields the correct position vector  $-7\mathbf{i} + 11\mathbf{j} - 19\mathbf{k}$ . A few candidates, by deducing that the intersection point is the midpoint of  $A$  and  $B$  were able to write down  $\frac{9+x}{2} = 1$ ,  $\frac{3+y}{2} = 7$  and  $\frac{13+z}{2} = -3$ , in order to find the position vector of  $B$ .

### Question 5

Nearly all candidates knew the method for solving part (a), although there were many errors in differentiating trig functions. In particular  $\frac{d}{dt}(2 \cos 2t)$  was often incorrect. It was clear from

both this question and question 2 that, for many, the calculus of trig functions was an area of weakness. Nearly all candidates were able to obtain an exact answer in surd form. In part (b), the majority of candidates were able to eliminate  $t$  but, in manipulating trigonometric identities, many errors, particularly with signs, were seen. The answer was given in a variety of forms and all exact equivalent answers to that printed in the mark scheme were accepted. The value of  $k$  was often omitted and it is possible that some simply overlooked this. Domain and range remains an unpopular topic and many did not attempt part (c). In this case, inspection of the printed figure gives the lower limit and was intended to give candidates a lead to identifying the upper limit.

### Question 6

In part (a), a majority of candidates were able to subtract the given position vectors correctly in order to find  $\overline{AB}$ . Common errors in this part included some candidates subtracting the position vector the wrong way round and a few candidates who could not deal with the double negative when finding the **k** component of  $\overline{AB}$ .

In part (b), a significant majority of candidates were able to state a vector equation of  $l_1$ . A significant number of these candidates, however, wrote 'Line = ' and omitted the 'r' on the left hand side of the vector equation, thereby losing one mark.

Many candidates were able to apply the dot product correctly in part (c) to find the correct angle. Common errors here included applying a dot product formula between  $\overrightarrow{OA}$  and  $\overrightarrow{OB}$ ; or applying the dot product between either  $\overrightarrow{OA}$  or  $\overrightarrow{OB}$  and the direction vector of  $l_1$ . Interestingly, a surprising number of candidates either simplified  $\sqrt{(1)^2 + (-2)^2 + (2)^2}$  to  $\sqrt{5}$  or when finding the dot product multiplied -2 by 0 to give -2.

Part (d) proved more discriminating. The majority of candidates realised that they needed to put the line  $l_1$  equal to line  $l_2$ . A significant number of these candidates, however, were unable to write  $l_2$  as  $\mu(\mathbf{i} + \mathbf{k})$  or used the same parameter (usually  $\lambda$ ) as they had used for  $l_1$ . Such candidates then found difficulty in making further progress with this part.

## Question 7

Part (a) was fully correct in the great majority of cases but the solutions were often unnecessarily long and nearly two pages of working were not unusual. The simplest method is to equate the  $\mathbf{j}$  components. This gives one equation in  $\lambda$ , leading to  $\lambda = 3$ , which can be substituted into the equation of  $l_1$  to give the coordinates of  $C$ . In practice, the majority of candidates found both  $\lambda$  and  $\mu$  and many proved that the lines were coincident at  $C$ . However the question gave the information that the lines meet at  $C$  and candidates had not been asked to prove this. This appeared to be another case where candidates answered the question that they had expected to be set, rather than the one that actually had been.

The great majority of candidates demonstrated, in part (b), that they knew how to find the angle between two vectors using a scalar product. However the use of the position vectors of  $A$  and  $B$ , instead of vectors in the directions of the lines was common. Candidates could have used either

the vectors  $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$  and  $\begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix}$ , given in the question, or  $\overrightarrow{AC}$  and  $\overrightarrow{BC}$ . The latter was much the

commoner choice but many made errors in signs. Comparatively few chose to use the cosine rule. In part (c), many continued with the position vectors they had used incorrectly in part (b) and so found the area of the triangle  $OAB$  rather than triangle  $ABC$ . The easiest method of completing part (c) was usually to use the formula  $\text{Area} = \frac{1}{2}ab\sin C$  and most chose this. Attempts to use  $\text{Area} = \frac{1}{2}\text{base} \times \text{height}$  were usually fallacious and often assumed that the triangle was isosceles. A few complicated attempts were seen which used vectors to find the coordinates of the foot of a perpendicular from a vertex to the opposite side. In principle, this is possible but, in this case, the calculations proved too difficult to carry out correctly under examination conditions.

### Question 8

Many candidates, who answered part (a), were able to separate the variables correctly and integrate both sides of their equation to obtain  $\ln P = kt$ . At this point a significant number of candidates either omitted the constant of integration or were unable to deal with the boundary conditions given in the question. Some candidates, for example, wrote down  $P = e^{kt} + c$ ; and stated that  $c = P_0$  to give the common incorrect solution of  $P = P_0 + e^{kt}$ . Other candidates used  $P_0$  instead of  $P$  in their attempts, and then struggled to find the constant of integration. Some candidates, who correctly evaluated the constant of integration, did not make  $P$  the subject of the equation but left their answer as  $\ln P = kt + \ln P_0$ .

Those candidates who had successfully answered part (a) were able to gain most of the marks available in part (b). A few of these candidates, however, struggled to convert the correct time in hours to the correct time in minutes. Those who did not progress well in part (a) may have gained only a method mark in part (b) by replacing  $P$  in their part (a) equation with  $2P_0$ .

Those candidates who were successful in the first two parts of this question usually succeeded to score most of the marks available in parts (c) and (d). In part (c) some candidates incorrectly integrated  $\lambda \cos \lambda t$ . In part (d), a significant number of candidates found difficulty in solving the equation  $\sin(2.5t) = \ln 2$ . It was not uncommon for some of these candidates to write  $t = \frac{\ln 2}{\sin(2.5)}$ . Also, in part (d), some candidates did not work in radians when evaluating  $t = \arcsin(\ln 2)$ .

### Statistics for C4 Practice Paper G3

Qu	Max score	Modal score	Mean %	Mean score for students achieving grade:							
				ALL	A*	A	B	C	D	E	U
1	5		78	3.88		4.59	4.06	3.64	3.00	2.30	1.38
2	6		64	3.81	5.84	5.13	4.00	2.69	1.71	0.94	0.36
3	4	1	57	2.26	3.53	2.49	2.09	1.73	1.58	1.50	1.15
4	13		61	7.94		10.15	7.19	4.59	3.25	1.74	0.58
5	10		54	5.38		7.41	5.34	3.97	2.72	1.64	0.63
6	11		57	6.30		8.66	5.80	4.15	3.11	1.68	1.27
7	12		54	6.42	10.86	8.23	6.15	4.39	3.01	2.02	1.02
8	14		36	5.09		8.99	3.90	1.81	0.80	0.35	0.09
	<b>75</b>		<b>55</b>	<b>41.08</b>		<b>55.65</b>	<b>38.53</b>	<b>26.97</b>	<b>19.18</b>	<b>12.17</b>	<b>6.48</b>